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THE ANALYSIS OF
SMALL WATER DISTRIBUTION SYSTEMS

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THE ANALYSIS OF
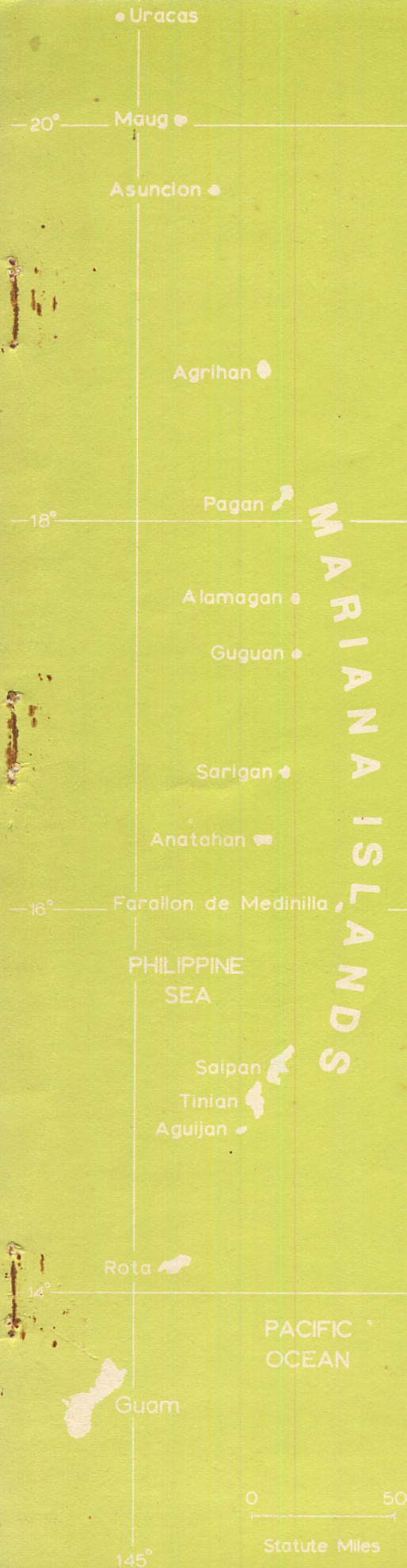# SMALL WATER DISTRIBUTION SYSTEMS

by

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UNIVERSITY OF GUAM
Water Resources Research Center
Technical Report No. 4

January 1978

Project Completion Report
for
A MODEL OF A PORTION OF THE
PUBLIC UTILITY AGENCY OF GUAM (PUAG) WATER DISTRIBUTION SYSTEM

ABSTRACT

A method is derived for determining the flow lost per foot of pipe in situations involving pipes having leaks or illegal connections. The flow into the pipe and the pressure drop over the length of the pipe must be known. An example problem is solved.

A modification of the Hardy-Cross technique of pipe network analysis is presented which accounts for flow out of the network per foot of pipe. This modification can be used to account for unintentional loss as well as outflow to consumers in situations where an accurate description of the flow is required. This situation is most likely to arise in small distribution systems where the outflow to each consumer may be a significant part of the total flow in a pipe. Two example problems are solved, the second involving an expansion of the network considered in the first.

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INTRODUCTION

For all practical purposes, present day water distribution systems serving the civilian community of Guam were constructed subsequent to WWII. Prior to 1950, the beginning of civilian government on Guam, water distribution, like other utilities, was handled by the Navy government. Subsequent to that date, water distribution has been under the jurisdiction of the Public Utility Agency of Guam (PUAG).

During the latter phases of WWII, after American forces reoccupied Guam, a period of reconstruction ensued along with the development of water distribution systems in some of the larger cities. This development was hasty and rule of thumb design practices were often used. Records do not exist concerning the location of all pipes presently comprising these systems. Following this period, development was more orderly and records are available. However, in general, system development (and the search for new sources of potable water) has been the result of response to demand rather than the result of a master plan. This type of development has led to poor system performance in many areas, characterized by low pressures and, at worst, periods of no flow.

The problem becomes compounded because large quantities of water are lost in the distribution system (i.e., exist at the source but do not reach the consumer). Estimates of up to 30%

loss in the system have been made. There are at least two ex-
planations for this. Firstly, leakage occurs, probably in older
pipes. It is even possible that open pipes may exist, dis-
charging their contents into the ground. Secondly, it is known
that numerous illegal unmetered connections exist. Most have
probably been made with equipment acquired illegally from PUAG.

OBJECTIVES

As originally planned, this project involved the construc-
tion of a physical model of a portion of the PUAG water distri-
bution system. Serious consideration was later given to the
possibility of constructing a digital computer model. However,
difficulties in securing the metering devices required for
obtaining field data and in locating a suitable portion of the
system for analysis prevented the accomplishment of the original
objectives.

As an alternative, the necessary theory has been developed
and examples provided which illustrate how these objectives can
be accomplished. Specifically, analytical procedures are pre-
sented whereby it is possible:

1. to locate areas of leakage and/or illegal con-
    nections, and

2. to determine the flow and pressure distribution
    within a portion of the system.

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SCOPE AND METHOD

In case 1, pressure readings must be taken at fire hydrants or other convenient locations. Flow measurements must also be taken at one or more locations. By using this data as input to equations that will be derived in the Theory section of the report, it is possible to compute the flow out of the system per unit length of pipe. This flow can be compared with consumer meter readings in an effort to locate areas of loss. A numerical example of this type is worked in the Applications section of the report.

In case 2, the flows into and out of the system must be known. These can be obtained from flow measurements taken in the field and consumer meter readings. By using this data as input to an iterative technique that will be derived in the Theory section of the report, it is possible to determine the flow and pressure distribution in the portion of the system being analyzed. An example of this type is worked in the Applications section of the report. Furthermore, this technique makes it possible to predict the effects of alterations (expansions) of the system on its performance (i.e., the given input and output flows are modified in accordance with the expected increase in demand and changes in system performance can be observed, particularly areas of low pressure). An example of this type is also worked in the Applications section of the report.

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The procedures which are presented are unique in that they account for all flow leaving pipes in a distribution system; outflows are not lumped together as is usually the case. This approach is especially valuable in the case of small distribution systems such as exist on Guam.

The procedures which are presented do not include provisions for pumping and/or storage. However, it will be seen that these items are simple to isolate from the portion of the system being considered.

THEORY

General Equations

The mathematical analysis of fluid flow in a network of pipes requires that three conditions be satisfied:

1. The algebraic sum of the flows, Q, at a junction must be zero.

2. The algebraic sum of the head losses, H, around a loop must be zero.

3. The proper relationship between head loss and flow must be satisfied for each pipe.

The first and second conditions are the well-known circuit laws and may be expressed as:

$$\Sigma Q = 0 \qquad \ldots \quad 1$$

$$\Sigma H = 0 \qquad \ldots \quad 2$$

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Numerous relationships expressing the third condition exist.
Because of simplicity, an exponential relationship is frequently
used:

$$H = kQ^n \qquad\qquad\qquad\qquad \dots \quad 3$$

where k is a constant that depends on pipe length, diameter, and
condition; n is also a constant, approximately equal to 1.85,
that depends on pipe condition. An alternate form of equation 3
is:

$$H = k' Q^n \ell \qquad\qquad\qquad\qquad \dots \quad 4$$

where ℓ is the length of the pipe being considered and k' is a con-
stant that is independent of pipe length. (i.e., $k' = k/\ell$.)

In case of small distribution systems, the flow to each con-
sumer may be a significant part of the total flow in a given pipe
of the network. Consequently, a method of accounting for all
flow from each pipe is desirable. One way of achieving this
objective is by means of a "distributed take-off" (similar to a
distributed load on a beam), q(x), having units of flow/length.
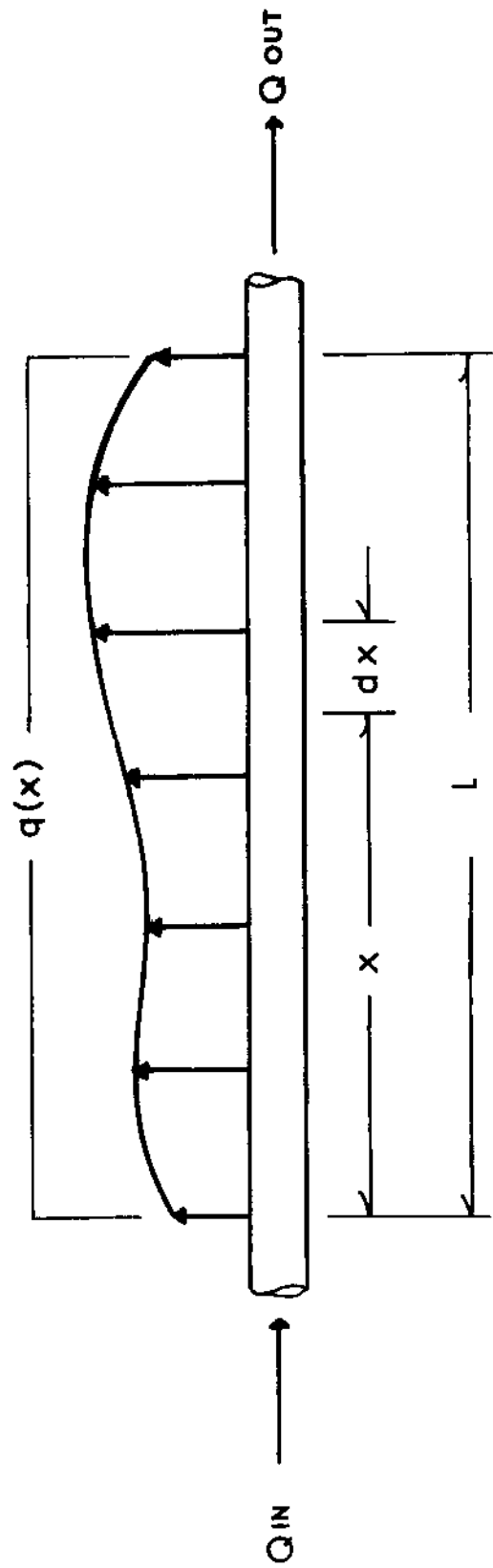This concept is illustrated in Figure 1.

FIGURE 1. PARAMETERS FOR DISTRIBUTED TAKE-OFF.

The flow, Q_x, at a point x in the pipe is

$$Q_x = Q_{in} - \int_0^x q(x)dx$$

or, assuming that q(x) is a constant (i.e., q(x) = q),

$$Q_x = Q_{in} - qx \qquad \qquad \ldots \quad 5$$

Q_{in} and Q_{out} are the flows into and out of the length of pipe, respectively. If the entire length of pipe is considered, then

$$Q_{out} = Q_{in} - q\ell \qquad \qquad \ldots \quad 6$$

Using equation 4, the head loss occurring in a length of pipe dx at position x is

$$d H' = k' Q_x^{\ n} dx$$

where the prime following H denotes head loss in a pipe having distributed take-off. If Q varies as indicated by equation 5, then

$$d H' = k' (Q_{in} - qx)^n dx$$

Integrating over the length of the pipe,

$$H' = k' \int_0^\ell (Q_{in} - qx)^n dx$$

$$= \frac{-k' (Q_{in} - qx)^{n+1}}{(n+1) q} \Bigg|_0^\ell$$

$$= \frac{k' \left[Q_{in}^{\ n+1} - (Q_{in} - q\ell)^{n+1} \right]}{(n+1) q} \qquad \qquad \ldots \quad 7$$

or, making use of equation 6,

$$H' = \frac{k'\left(Q_{in}^{n+1} - Q_{out}^{n+1}\right)}{(n+1)\, q} \qquad \cdots \quad 8$$

Equation 7 or 8 gives the head loss in a length of pipe having distributed take-off.

As q approaches zero, it is observed that equation 8 becomes indeterminate (i.e., approaches 0/0). Using l'Hopital's rule,

$$\underset{q \to o}{\text{limit}}\ H' = \left. \frac{k'}{n+1}\ \frac{\frac{d}{dq}\left[Q_{in}^{n+1} - (Q_{in} - q\ell)^{n+1}\right]}{\frac{d}{dq}\, q} \right|_{q \to o}$$

$$= \left. \frac{k'}{n+1}\left[- (n+1)(Q_{in} - q\ell)^{n}(-\ell)\right] \right|_{q \to o}$$

$$= k'\, Q_{in}^{\ n}\, \ell$$

which is equivalent to equation 4. Thus, as expected, the head loss in a pipe having distributed take-off reduces to the case of a conventional pipe when q approaches zero. This is the maximum head loss that can occur in a pipe having distributed take-off since all the water must travel the full length of the pipe. Making note of this, the previous equation can be rewritten as

$$H'_{max} = k'\, Q_{in}^{\ n}\, \ell \qquad \cdots \quad 9$$

8

The minimum head loss occurs when the outflow, Q_{out}, approaches zero (i.e., none of the water travels the full length of the pipe) and q is, therefore, maximum. Using equation 6,

$$Q_{out} = Q_{in} - q_{max} \, \ell = 0$$

or,

$$q_{max} = Q_{in}/\ell \qquad\qquad \ldots \quad 10$$

Substituting this result into equation 8,

$$H'_{min} = \frac{k' \left[Q_{in}^{n+1} - (Q_{in} - (Q_{in}/\ell) \, \ell)^{n+1} \right]}{(n+1) \, (Q_{in}/\ell)}$$

$$= \frac{k' \, Q_{in}^{n} \, \ell}{n+1} \qquad\qquad \ldots \quad 11$$

In summary, the lower limit on q, q = 0, yields the upper limit on H', equation 9; the upper limit on q, q = $Q_{in}/1$, yields the lower limit on H', equation 11.

The Hardy-Cross technique of pipe network analysis, employing the method of balancing heads[1], can be adapted to systems having lines with distributed take-off, such as that illustrated in Figure 2.

[1]Fair, G. M., Geyer, J. C., Okun, D. A., Water and Wastewater Engineering Volume 1. Water Supply and Wastewater Removal, John Wiley and Sons, Inc., New York, 1966, p. 13-16.
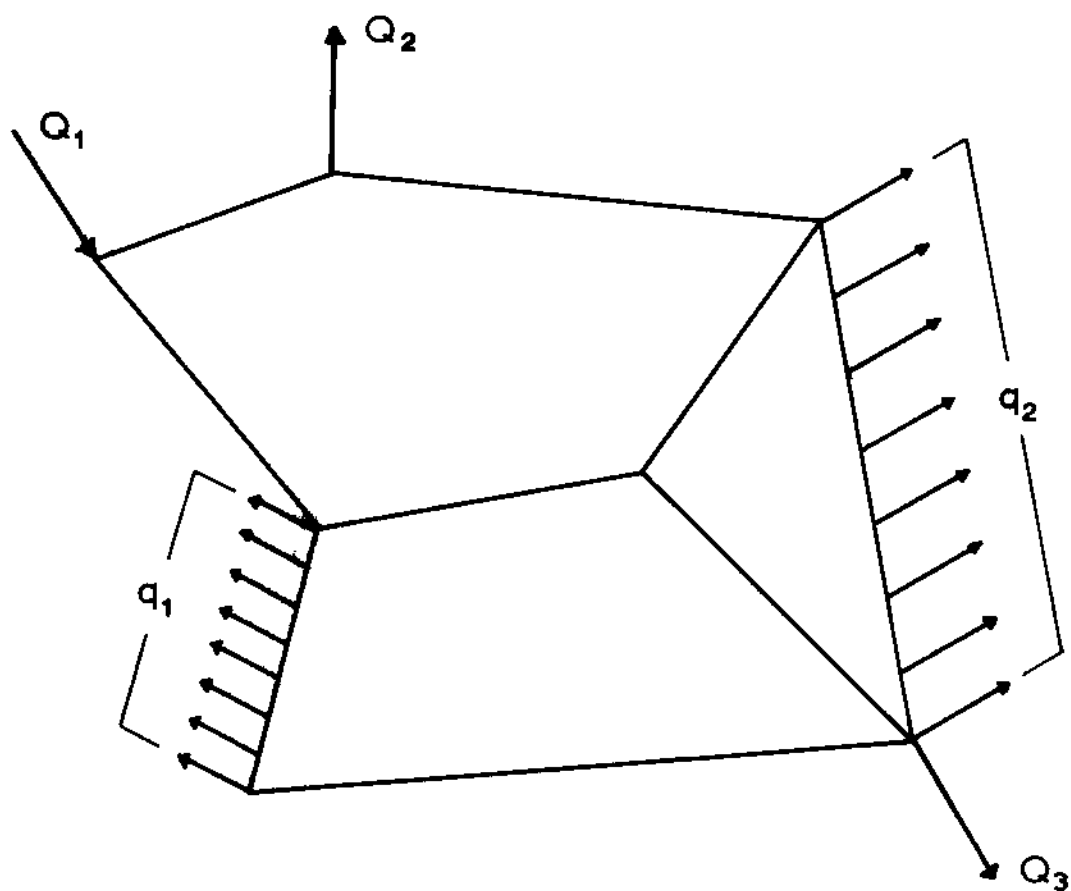
FIGURE 2. DISTRIBUTION SYSTEM INCLUDING LINES WITH DISTRIBUTED
TAKE-OFF.

In this method, the flows, Q, entering and leaving the system and
the distributed take-offs, q, are assumed to be known from field
data. Flows are initially assumed in each loop such that $\Sigma Q = 0$
at each junction. These flows do not necessarily yield head losses
such that $\Sigma H = 0$ around a loop. In each loop a small counterflow,
ΔQ, is therefore introduced such that $\Sigma H = 0$ around the loop. Using
equations 3 and 8,

$$\Sigma H = \Sigma k \ (Q+\Delta Q)^n + \frac{1}{n+1} \Sigma \frac{k'}{q} \ \left[(Q_{in}+\Delta Q)^{n+1} - (Q_{out}+\Delta Q)^{n+1} \right] = 0$$

where flow both in pipes having distributed take-off and in con-
ventional pipes has been accounted for. Expanding each power series
and neglecting terms involving ΔQ^2 and higher order quantities,

$$\Sigma kQ^n + n \ \Sigma k \ (\Delta Q)Q^{n-1} + \ldots$$

$$+ \frac{1}{n+1} \Sigma \frac{k'}{q} \ Q_{in}^{n+1} + \Sigma \frac{k'}{q} \ (\Delta Q)Q_{in}^{n} + \ldots$$

$$- \frac{1}{n+1} \Sigma \frac{k'}{q} \ Q_{out}^{n+1} - \Sigma \frac{k'}{q} \ (\Delta Q)Q_{out}^{n} - \ldots = 0$$

Solving for ΔQ,

$$\Delta Q = - \frac{\Sigma kQ^n + \frac{1}{n+1} \Sigma \frac{k'}{q} \left[Q_{in}^{n+1} - Q_{out}^{n+1} \right]}{n\Sigma kQ^{n-1} + \Sigma \frac{k'}{q} (Q_{in}^{n} - Q_{out}^{n})} \qquad \ldots \quad 12$$

or, making use of equations 3 and 9,

$$\Delta Q = - \frac{\Sigma H + \Sigma H'}{n\Sigma \frac{H}{Q} + \Sigma H''} \qquad \ldots \quad 13$$

where

$$\Sigma H'' = \Sigma \frac{k'}{q} \left(Q_{in}^{\ n} - Q_{out}^{\ n} \right) \qquad \cdots \quad 14$$

This term has no physical meaning whereas the other terms in equation 13 do.

As in the traditional Hardy-Cross method, the corrections, ΔQ, yield new values of flow for the various pipes in the network. The procedure continues in an iterative fashion until the desired degree of accuracy is achieved, the result being the correct value of flow into and out of each junction as well as the head loss in each pipe.

Hazen-Williams Equation

The Hazen-Williams equation[2] is an example of the exponential relationship between head loss and flow in a pipe given by equation 3

$$H = 1.5 \times 10^{-5} \left(\frac{Q}{C} \right)^{1.85} \frac{\ell}{d^{4.87}} \qquad \cdots \quad 15$$

where k of equation 3 is

$$k = \frac{1.5 \times 10^{-5} \ell}{C^{1.85} d^{4.87}} \qquad \cdots \quad 16$$

The following units have been used

H = head loss (ft)

Q = flow (gal/day)

ℓ = length (ft)

d = diameter (in)

C = coefficient of hydraulic capacity (determined from field tests).

[2]ibid, p. 12-8 .

The Hazen-Williams approach can be used in other previously derived equations by noting that

$$k' = \frac{k}{\ell} = \frac{1.5 \times 10^{-5}}{C^{1.85} d^{4.87}} \qquad \dots \quad 17$$

and setting n equal to 1.85.

APPLICATIONS

Loss Detection

Given: Pipe having distributed take-off and

ℓ = 1,000 ft.

d = 3 in.

Q_{in} = 100,000 gal/day

C = 100

Find: q for a pressure drop of 8 psi over the length of the pipe.

Solution: q can be determined from equation 7 by means of the iterative procedure presented in the flow diagram in Appendix 1. A Fortran computer program which utilizes this procedure is also given in Appendix 1. The answer to this problem is q = 32.0 gal/day/ft.

Distribution System Analysis

Given: Network shown in Figure 3 and C = 100.

Find: Flow and pressure distribution within the network.

Solution: With the aid of equation 13, the Hardy-Cross iterative technique can be used to solve this problem. A Fortran computer
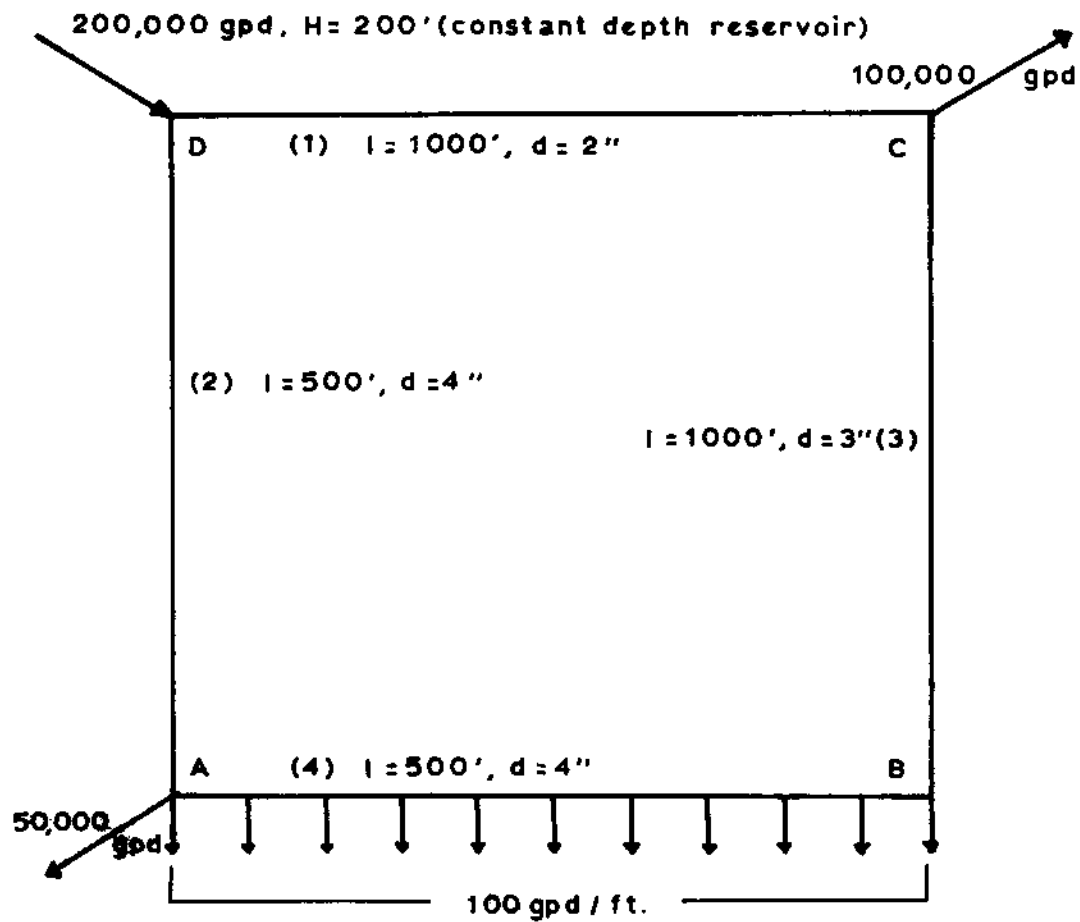
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FIGURE 3. EXAMPLE PROBLEM: DISTRIBUTION SYSTEM ANALYSIS.

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program which utilizes this technique is given in Appendix 2. The following flows were assumed:

Q_1 = 175,000 gal/day

Q_2 = -25,000 gal/day

Q_3 = 75,000 gal/day

$Q_{4\ in}$ = 75,000 gal/day, $Q_{4\ out}$ = 25,000 gal/day

where the subscript denotes pipe number and clockwise flows are positive. Five iterations yield a correction, ΔQ, of less than 10 gal/day. The rate of covergence is quite rapid even though the assumed flows were very different from the final ones. The results of the five iterations are given in Table 1. The final flows and head losses are:

Q_1 = 32,700 gal/day, H_1 = 23.0 ft

Q_2 = -167,000 gal/day, H_2 = -8.07 ft

Q_3 = -67,300 gal/day, H_3 = -12.2 ft

Q_4 = -67,300 gal/day, Q_{out} = -117,000 gal/day, H_4' = -2.74 ft

The final heads at each junction are:

H_D = 200 ft (86.7 psig)

H_A = 200-8.07 = 192 ft (83.2 psig)

H_B = 192-2.74 = 189 ft (81.9 psig)

H_C = 189-12.2 = 177 ft (76.7 psig)

or, H_C = 200-23.0 = 177 ft (76.7 psig)

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TABLE 1. RESULTS OF HARDY-CROSS ITERATIVE TECHNIQUE FOR DIS-
TRIBUTION SYSTEM ANALYSIS PROBLEM.

ITERATION NO. 1

PIPE NO	LENGTH	DIAMETER	Q	QOI	QOO	H	H/Q	H'	H''
1	0.100E+04	0.200E+01	0.175E+06	0.000E+00	0.000E+00	0.513E+03	0.293E-02	0.000E+00	0.000E+00
2	0.500E+03	0.400E+01	-0.250E+05	0.000E+00	0.000E+00	-0.239E+00	0.958E-05	0.000E+00	0.000E+00
3	0.100E+04	0.300E+01	0.750E+05	0.000E+00	0.000E+00	0.148E+02	0.198E-03	0.000E+00	0.000E+00
4	0.500E+03	0.400E+01	0.000E+00	0.750E+05	0.250E+05	0.000E+00	0.000E+00	0.920E+00	0.313E-04

DQ = -0.935E+05

ITERATION NO. 2

PIPE NO	LENGTH	DIAMETER	Q	QOI	QOO	H	H/Q	H'	H''
1	0.100E+04	0.200E+01	0.345E+05	0.000E+00	0.000E+00	0.133E+03	0.158E-02	0.000E+00	0.000E+00
2	0.500E+03	0.400E+01	-0.116E+06	0.000E+00	0.000E+00	-0.406E+01	0.352E-04	0.000E+00	0.000E+00
3	0.100E+04	0.300E+01	-0.155E+05	0.000E+00	0.000E+00	-0.804E+00	0.518E-04	0.000E+00	0.000E+00
4	0.500E+03	0.400E+01	0.000E+00	-0.155E+05	-0.655E+05	0.000E+00	0.000E+00	-0.644E+00	0.265E-04

DQ = -0.411E+05

ITERATION NO. 3

PIPE NO	LENGTH	DIAMETER	Q	QOI	QOO	H	H/Q	H'	H''
1	0.100E+04	0.200E+01	0.433E+05	0.000E+00	0.000E+00	0.388E+02	0.894E-03	0.000E+00	0.000E+00
2	0.500E+03	0.400E+01	-0.157E+06	0.000E+00	0.000E+00	-0.714E+01	0.456E-04	0.000E+00	0.000E+00
3	0.100E+04	0.300E+01	-0.567E+05	0.000E+00	0.000E+00	-0.883E+01	0.156E-03	0.000E+00	0.000E+00
4	0.500E+03	0.400E+01	0.000E+00	-0.567E+05	-0.107E+06	0.000E+00	0.000E+00	-0.219E+01	0.484E-04

DQ = -0.393E+04

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ITERATION NO. 4

PIPE NO	LENGTH	DIAMETER	Q	QDI	QDO	H	H/Q	H'	H''
1	0.100E+04	0.200E+01	0.334E+05	0.000E+00	0.000E+00	0.240E+02	0.717E-03	0.000E+00	0.000E+00
2	0.500E+03	0.400E+01	-0.167E+06	0.000E+00	0.000E+00	-0.800E+01	0.430E-04	0.000E+00	0.000E+00
3	0.100E+04	0.300E+01	-0.666E+05	0.000E+00	0.000E+00	-0.119E+02	0.179E-03	0.000E+00	0.000E+00
4	0.500E+03	0.400E+01	0.000E+00	-0.666E+05	-0.117E+06	0.000E+00	0.000E+00	-0.270E+01	0.533E-04

DQ = -0.756E+03

ITERATION NO. 5

PIPE NO	LENGTH	DIAMETER	Q	QDI	QDO	H	H/Q	H'	H''
1	0.100E+04	0.200E+01	0.327E+05	0.000E+00	0.000E+00	0.230E+02	0.703E-03	0.000E+00	0.000E+00
2	0.500E+03	0.400E+01	-0.167E+06	0.000E+00	0.000E+00	-0.807E+01	0.482E-04	0.000E+00	0.000E+00
3	0.100E+04	0.300E+01	-0.673E+05	0.000E+00	0.000E+00	-0.122E+02	0.181E-03	0.000E+00	0.000E+00
4	0.500E+03	0.400E+01	0.000E+00	-0.673E+05	-0.117E+06	0.000E+00	0.000E+00	-0.274E+01	0.537E-04

DQ = -0.462E+01

Distribution System Modification

Given: Network shown in Figure 4 and C = 100 (note that this net-
work is the same as that of the previous problem except that a dis-
tributed take-off has been added to pipe 3 and the input flow in-
creased accordingly).

Find: Flow and pressure distribution within the network.

Solution: The method of solution is the same as that of the previous
problem. The following flows were assumed:

Q_1 = 50,000 gal/day

Q_2 = -250,000 gal/day

$Q_{3\ in}$ = -50,000 gal/day, $Q_{3\ out}$ = -150,000 gal/day

$Q_{4\ in}$ = -150,000 gal/day, $Q_{4\ out}$ = -200,000 gal/day

In this case, because the assumed flows closely approximate the final
ones, only two iterations are required to achieve a correction, ΔQ,
of less than 10 gal/day. The results of the two iterations are given
in Table 2. The final flows and head losses are:

Q_1 = 50,900 gal/day, H_1 = 52.1 ft

Q_2 = -249,000 gal/day, H_2 = -16.8 ft

$Q_{3\ in}$ = -49,100 gal/day, $Q_{3\ out}$ = -149,000 gal/day, H_3' = -26.5 ft

$Q_{4\ in}$ = -149,000 gal/day, $Q_{4\ out}$ = -199,000 gal/day, H_4' = -8.73 ft

The final heads at each junction are:

H_D = 200 ft (86.7 psig)

H_A = 200 - 16.8 = 183 ft (79.3 psig)

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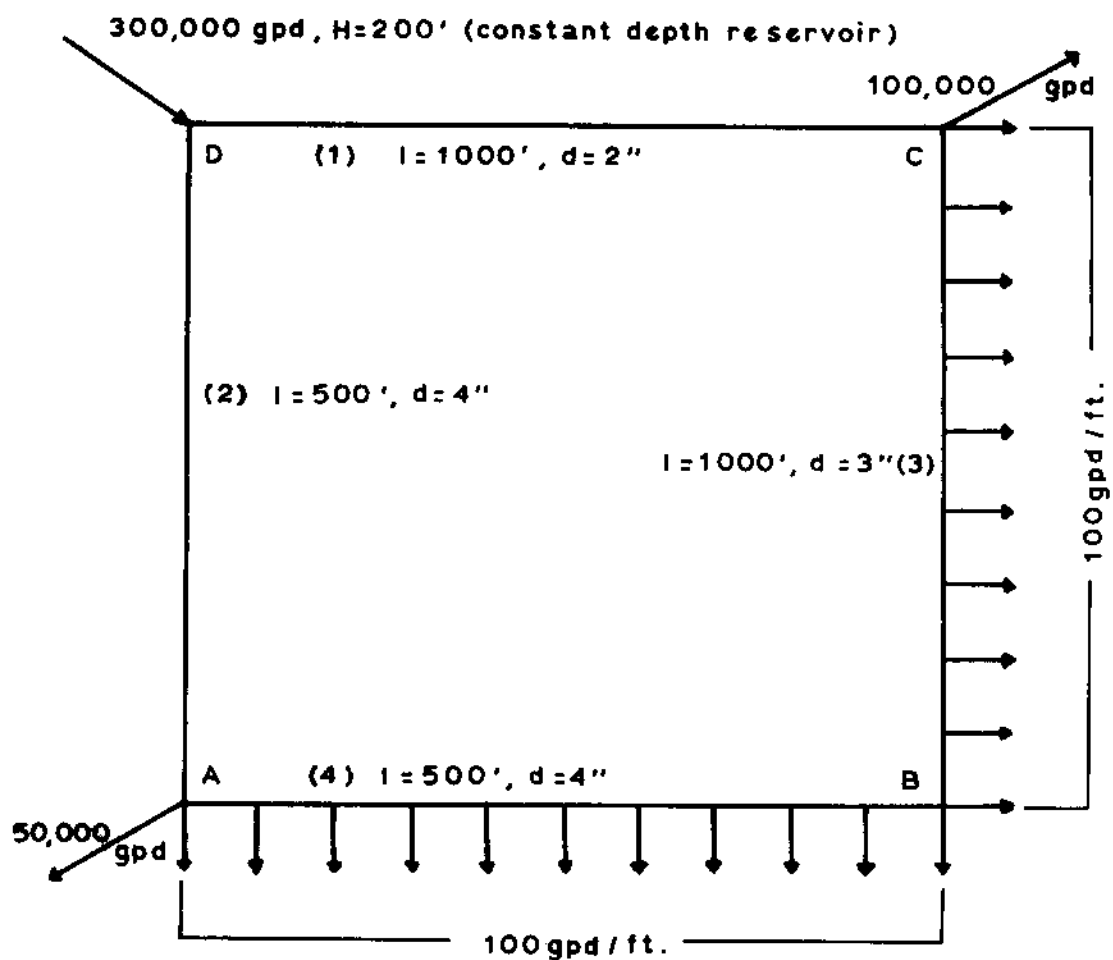FIGURE 4. EXAMPLE PROBLEM: DISTRIBUTION SYSTEM MODIFICATION.

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$$H_B = 183 - 8.73 = 174.5 \text{ ft (75.6 psig)}$$

$$H_C = 174.5 - 26.5 = 148 \text{ ft (64.1 psig)}$$

$$\text{or,} \quad H_C = 200 - 52.1 = 148 \text{ ft (64.1 psig)}$$

Note that, although the head at junction C is still adequate for household use, it has decreased significantly because of the increased flow requirements in the network.

TABLE 2.   RESULTS OF HARDY-CROSS ITERATIVE TECHNIQUE FOR DIS-
TRIBUTION SYSTEM MODIFICATION PROBLEM.

ITERATION NO. 1

PIPE NO	LENGTH	DIAMETER	Q	QDI	QOO	H	H/Q	H'	H''
1	0.100E+04	0.200E+01	0.500E+05	0.000E+00	0.000E+00	0.505E+02	0.101E-02	0.000E+00	0.000E+00
2	0.500E+03	0.400E+01	-0.250E+06	0.000E+00	0.000E+00	-0.170E+02	0.678E-04	0.000E+00	0.000E+00
3	0.100E+04	0.300E+01	0.000E+00	-0.500E+05	-0.150E+06	0.000E+00	0.000E+00	-0.267E+02	0.465E-03
4	0.500E+03	0.400E+01	0.000E+00	-0.150E+06	-0.200E+06	0.000E+00	0.000E+00	-0.881E+01	0.926E-04

DQ =   0.862E+03

ITERATION NO. 2

PIPE NO	LENGTH	DIAMETER	Q	QDI	QOO	H	H/Q	H'	H''
1	0.100E+04	0.200E+01	0.509E+05	0.000E+00	0.000E+00	0.521E+02	0.102E-02	0.000E+00	0.000E+00
2	0.500E+03	0.400E+01	-0.249E+06	0.000E+00	0.000E+00	-0.168E+02	0.676E-04	0.000E+00	0.000E+00
3	0.100E+04	0.300E+01	0.000E+00	-0.491E+05	-0.149E+06	0.000E+00	0.000E+00	-0.265E+02	0.461E-03
4	0.500E+03	0.400E+01	0.000E+00	-0.149E+06	-0.199E+06	0.000E+00	0.000E+00	-0.873E+01	0.922E-04

DQ = -0.393E+01

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Flow Diagram for Solution of Loss Detection Problem

Read: ℓ, d, Q_{in}, C, Δp, n

$q_{max} = Q_{in}/\ell$

$q_{min} = 0$ — eq 10

$H' = \dfrac{\Delta p\ (144)}{62.4}$

$k' = \dfrac{1.5 \times 10^{-5}}{C^n\ d^{4.87}}$ — eq 18

$q = \dfrac{q_{max} - q_{min}}{2}$

$H'_{test} = k' \dfrac{\left[Q_{in}^{n+1} - (Q_{in} - q\ell)^{n+1}\right]}{(n+1)\ Q}$ — eq 7

$\Delta = H'_{test} - H'$

IF $|\Delta| \leq 0.1$ — Yes → Write: q

No

IF Δ

$q_{min} = q$ (+) $q_{max} = q$ (−)

0 → Write: q
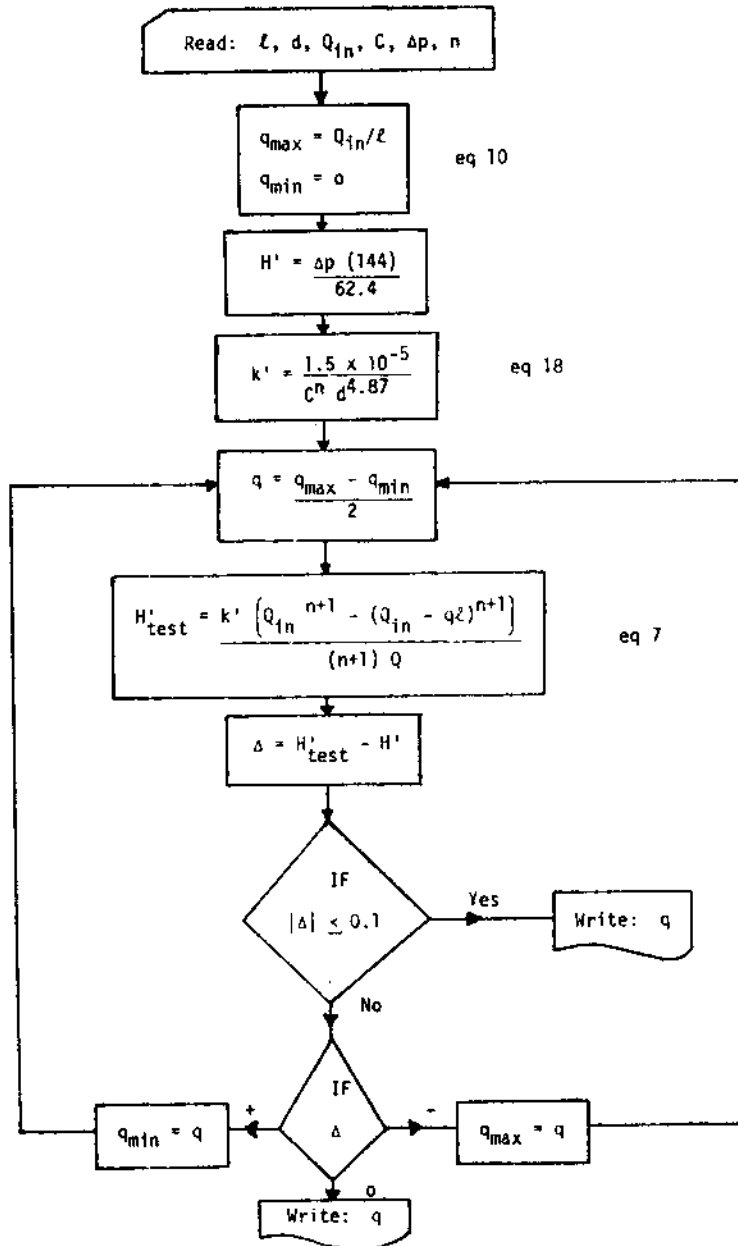
Fortran Computer Program for Loss Detection

```
      REAL L,N,KP
      READ(9,1)L,D,QIN,C,DELTAP,N
1     FORMAT(6F10.0)
      QMAX=QIN/L
      QMIN=0.
      HP=DELTAP*144./62.4
      KP=1.5E-5/(C**N*D**4.87)
6     Q=(QMAX+QMIN)/2.
      HPTEST=KP*(QIN**(N+1.)-(QIN-Q*L)**(N+1.))/((N+1.)*Q)
      DELTA=HPTEST-HP
      IF(ABS(DELTA).LE.0.1)GO TO 2
      IF(DELTA)4,2,5
4     QMAX=Q
      GO TO 6
5     QMIN=Q
      GO TO 6
2     WRITE(3,3)Q
3     FORMAT(//1X,'Q =',F5.1)
      STOP
      END
```

APPENDIX 2

Fortran Computer Program for Distribution System Analysis

```
      DIMENSION Q(L),AL(4),D(10),QDI(10),QDD(10),QD(10),HP(10),DHDP(10
     1)
      READ (9,10) (Q(I),I=1,4)
      READ (9,10) (AL(I),I=1,4)
      READ (9,10) (D(I),I=1,4)
      READ (9,10) (QD(I),I=1,4)
      READ (9,10) (QDI(I),I=1,4)
      READ (9,10) (QDD(I),I=1,4)
   10 FORMAT(4F12.2)
      C=100.
      A=1.5E-05
      EN=1.85
      CEN=C**EN
      DO 700 L=1,13
      SHQ=0.
      SH=0.
      SHP=0.
      SHDP=0.
      WRITE(3,50) L
   50 FORMAT(17H1    ITERATION NO.,I3)
      WRITE(3,60)
   60 FORMAT(//112H  PIPE NO   LENGTH    DIAMETER        Q        QDI
     1     QDD        H       H/Q       H'       H'')
      DO 500 J=1,4
      IP=J
      IF (Q(J))100,200,100
  100 AK=A*AL(J)/(CEN*D(J)**4.87)
      IF (Q(J)) 120,110,110
  110 HQ=AK*Q(J)**EN
      GO TO 150
  120 QQ=-Q(J)
      HQ=-AK*QQ**EN
  150 HQ=HQ/Q(J)
      SH=SH+HQ
      SHQ=SHQ+HQ
      GO TO 270
  200 BK=A/(CEN*D(J)**4.87)
      HQ=0.
      HQ=0.
      IF (QDI(J)) 240,230,230
  230 HDPI=BK*QDI(J)**EN/QD(J)
      HP1=HDPI*QDI(J)
      IF (QDD(J)) 242,232,232
  232 HDPD=BK*QDD(J)**EN/QD(J)
      HP2=HDPD*QDD(J)
      HP(J)=(HP1-HP2)/2.85
      GO TO 245
  240 QDIN=-QDI(J)
      HDPI=-BK*QDIN**EN/QD(J)
      HP1=HDPI*QDIN
  242 QDDT=-QDD(J)
      HDPD=-BK*QDDT**EN/QD(J)
      HP2=HDPD*QDDT
      HP(J)=(HP2-HP1)/2.85
  245 DHDP(J)=HDPI-HDPD
      SHP=SHP+HP(J)
      SHDP=SHDP+DHDP(J)
  270 WRITE(3,300)IP,AL(J),D(J),Q(J),QDI(J),QDD(J),HQ,HQ,HP(J),DHDP(J)
  300 FORMAT(//I6,9E12.3)
  500 CONTINUE
      QQ=-1.00*(SH+SHP)/(EN*SHQ+SHDP)
      WRITE(3,600)QQ
  600 FORMAT(////10H    QQ =   ,E10.3)
      DO 700 K=1,4
      IF (Q(K))610,650,610
  610 Q(K)=Q(K)+QQ
      GO TO 700
  650 QDI(K)=QDI(K)+QQ
      QDD(K)=QDD(K)+QQ
  700 CONTINUE
      STOP
      END
```